

# COSTING THE CONVERTS

Current evaluation methods for convertibles cannot cope with complicated features, say Water Cheung and Izzy Nelken. The authors propose a quadro tree pricing model which overcomes these shortcomings

Convertible bonds are particularly appealing in this time of sporadic economic recovery, as they allow investors to preserve their capital while taking advantage of rising share prices.

At the holder's option, the bonds can be converted into a certain pre-specified amount of the issuing company's common stock. If the common stock rises, so will the convertible's price. If the stock price falls, the investors will not suffer too much as they continue to receive interest payments.

Many convertibles also have early redemption features, such as an embedded call option, sinking fund or put option. Call features allow issuers to redeem the bond before it matures, within certain call protection provisions.

Absolute call protection prohibits the issuer from calling the bond before a certain date. Provisional call protection prevents the issuer from calling the bond unless the underlying stock is trading at a certain premium above the conversion price. If the issuer calls the bond while the stock is trading at a premium, the investors will convert to common stock. Thus, the issuer's option is to force conversion into stock.

## Previous pricing methods

Although these bonds have been around for a long time, there is no standard way of pricing vanilla convertibles or those with embedded options. The traditional methods outlined below all have their shortcomings so we have developed a more flexible and accurate valuation system.

## BREAK-EVEN PERIOD ANALYSIS

This compares two alternatives: owning shares versus owning convertibles. It first computes the ongoing *yield advantage*, coupon rate minus dividend rate, and then calculates the time required by the yield differential to compensate fully for the initial *conversion premium*. This is explained on page 376 in Fabozzi (1993).

This method serves only as a reasonable check on convertible securities. It does not take into account future dividend growth, certainty of dividend payment, option time value, or stock price volatility, nor does it generate a specific theoretical value. It cannot handle additional call or put features.

## DISCOUNT CASHFLOW ANALYSIS

This is also based on yield advantage but goes further and projects future cashflow - coupon income minus dividend income, with the possibility of incorporating dividend growth. The net present value of the projected cashflow is added to the current stock price. The sum is then multiplied by the conversion ratio to arrive at a "theoretical" convertible value.

Again, this method ignores the option time value and underlying stock price volatility and cannot handle additional call or put features. But it is better than break-even period analysis because it actually prices a convertible security.

This approach is more applicable to evaluating the convertible as a debt rather than as an equity instrument.

## SYNTHETICS (BOND + EQUITY OPTION)

This approach assumes the value of a convertible is equal to the sum of two components: a straight bond of the same coupon and maturity and a call option on the stock with the same exercise price.

The bond component, in the absence of a comparable bond with the same coupon and maturity, can be valued by discounting all future coupons and the maturing principal by the appropriate risk-adjusted discount curve. The option will be valued independently with a long-term option model (see Fabozzi).

However, a convertible is not equal to the sum of a straight bond and a call option on stock.

There is a major difference between a straight, freely-traded equity option which is exercisable by paying a known, fixed exercise price, and a convertible's converting feature, which is exercisable by turning in the value of a bond, whose value at any point in time depends on the value of the future cashflow relative to the current yield curve.

This method is more sophisticated than the other two, because it handles the option feature separately. Although incorrect in valuation, it does try to account for the option time value and volatility.

## LYON PRICING

McConnell and Schwartz (1986) have studied a Liquid Yield Option Note, which is a zero-coupon, convertible, callable and puttable

bond. They present the solution as a differential equation with a number of boundary conditions.

The major difficulty with this model is that it assumes a constant interest rate. However, it is well known that the term structure of interest rates is far from flat.

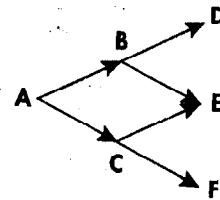
## Quadro tree approach

The authors have constructed a new convertible pricing model based on the concept of quadro-tree - a combination of two binary trees, an interest rate binomial tree and a stock prices binomial tree. In this fashion, we model the price's dependency on the underlying stock price as well as on the prevailing interest rates.

## Interest rate tree

A popular method of pricing a simple callable bond is to construct a tree of one-period forward interest rates. We adopt the model

### The interest rate tree



Node A: the one-year spot rate  
 Nodes B & C: possible one-year forward rates one year from today  
 Nodes D, E & F: possible one-year forward rates two years from today

developed by Kalotay, Williams and Fabozzi (1993).

The tree begins today. Each node is assigned the one-year (or period) forward rate beginning at that time.

Note that only the present is determined. We know that the information at node A is the one-year spot rate. However, since we are not sure of what the future will bring, there are a few possibilities. For example, nodes B and C represent two possible one-year rates which

may prevail one year from now. They are both equally likely and so have equal probabilities in the tree. The further we go out into the future, the more possibilities we have. Hence we have one possible value for the current rate, two possible values for the one-year rate one year from now, three possibilities for the rate two years from now, and so on.

Given that the volatility of the one-year rate is  $\sigma$ , and assume that the rate at C is  $r_1$ , then the rate at B is  $r_1 e^{2\sigma\Delta t}$  (in our case  $\Delta t = 1$ ). We set the rate at node A to be equal to the one-year benchmark yield.

We choose  $r_1$  so as to solve for the value of a two-year benchmark bond.

This is done as follows. Suppose the two-year benchmark bond has a coupon of 10% and is priced at P. For simplicity, and without loss of generality, we assume an annual pay bond. Let's guess  $r_1 = 7\%$ .

Two years from now, the bond will mature at par and will also pay out its coupon. Assume that one year from now, the one-year rate will be  $r_1 e^{2\sigma\Delta t}$ . We discount the maturity value of the bond, which is \$110.00 by that rate, and obtain the price of the bond one year from today. We repeat the same procedure but this time we assume that we are in node C and the one-year rate is  $r_1$ . We now have two possible prices for the benchmark one year from today.

However, the price of the bond today is its expected price one year from now plus the coupon payment, discounted back to today. Since we know the rate at node A, we can compute the average price of the bond at nodes B and C, add the coupon and discount it by the rate at node A. The price should be equal to the current market price of the two year benchmark. If it is not, we should adjust our guess for  $r_1$  until we obtain a match.

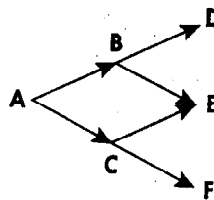
We then set the rate at F to  $r_2$ , the rate at E is  $r_2 e^{2\sigma\Delta t}$  and the rate at D is  $r_2 e^{4\sigma\Delta t}$ . We choose  $r_2$  so as to solve for the value of a three-year benchmark bond. We continue in this fashion until the entire tree is built.

It is assumed that there is an equal probability for an up move or a down move.

### Stock price tree

Hull (1993) has described how to build a tree for stock prices in the future. The actual model is attributed to Cox, Ross and Rubinstein (1979).

### The stock price tree



Node A: today's spot price  
 Nodes B & C: possible spot prices one year from today  
 Nodes D, E & F: possible stock prices two years from today

Since we are not sure of what the future will bring, there are a few possibilities.

The price at A is S. The price at B is Su and at C is Sd. We assume that the probability of a move to B is p and of a move to C is (1-p). The price at D is Suu, the price at E is S, the price at F is Sdd. u, d and p are easily calculated given the expected return of the stock  $\mu$ , the volatility of the stock price  $\sigma$  and the dividend rate q:

$$u = e^{\sigma\Delta t}$$

$$d = e^{-\sigma\Delta t}$$

$$p = (e^{(\mu-q)\Delta t} - d) / (u - d)$$

Now, using the risk neutrality argument, we can set  $\mu = r$ , the risk-free rate.

### A quadro tree

We will now combine the two approaches into a single "quadro-tree". Each node will have four descendants.

Out of each node A, we will have four descendants: B, C, D and E.

- B: Stock up, rates up
- C: Stock up, rates down
- D: Stock down, rates up
- E: Stock down, rates down

The price at maturity is 100. Then, when we move backwards in time, the instrument's price at A is computed as follows:

$$A = Z(0.5*p*B + 0.5*p*C + 0.5*(1-p)*D + 0.5*(1-p)*E + \text{coupon})$$

where Z is the discount factor:  $Z = e^{-r\Delta t}$  and r is the interest rate at node A. We now subject the price at A to the following logic:

1) Define  $R = \max(\text{call price}, N * \text{stock price})$  where N is the number of stocks into which the bond is convertible. If the stock price is greater than the boundary at which the issuer may call and the instrument price is greater than R, then set instrument price = R.

If the issuer intends to call the instrument at the call price then the holder gets the opportunity to convert to common shares. In most cases, the value of the shares is greater than the call price and so the issuer may "force conversion" on the holder.

2) If the instrument price is less than N\*stock price, then set instrument price = N\*stock price. This is because the value of the option cannot be smaller than its intrinsic value.

3) In all other cases, do not change the price at A.

Of course, other features of the bond (such as put options) may be modelled in this environment in a similar fashion. If the call and put features occur simultaneously, priority is given according to the prospectus.

### Case study

We used our model to price the bond of Abitibi-Price, which is both callable and convertible. It has a coupon of 7.85% and matures on March 1, 2003. It is callable by the issuer at certain pre-determined dates and prices.

However, this option is allowed on only if the price of the common stock of the issuer rises above a certain barrier. This type of bond is also convertible any time at the holder's option into a certain pre-determined number of common shares.

Of course, these benefits come at a price. We can assume that the price of a convertible is higher than the price of a similar regular bond. To make matters more complicated, the call option also has an impact on the price.

We used our quadro model to compute the "fair value" of this complicated instrument.

The bond has two embedded options.

- **Call option:** If the common stock of the issuer is priced above a certain barrier, the issuer may call the bond at par. The barrier may change as maturity approaches.

- **Conversion option:** The holder may convert the bond into a pre-determined number of common stocks at any time. The number of

common stocks to which the bond can be converted may change as maturity approaches. There are no restrictions on this right of the holder.

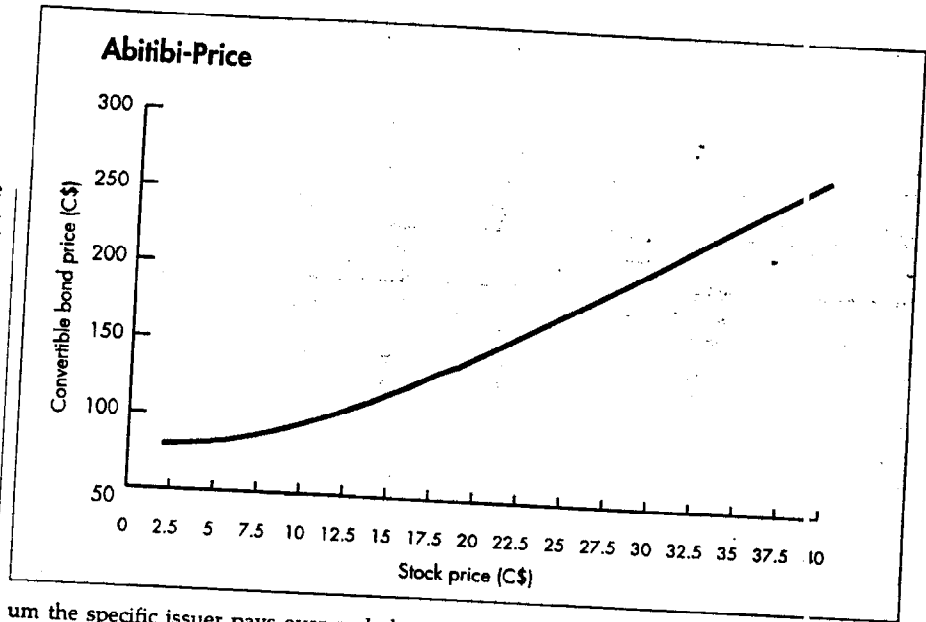
It may happen that the issuer will notify the holder of its intention to call the bond. In that case, the holder might choose to convert. Thus, we may say that the issuer has the power to force the investor to convert. This is known as "forced conversion".

**Assumptions**

We always assume that both the issuer and the holder will follow a logical course of action when choosing whether to exercise their respective options. Further, that the issuer and the holders are aware of each other and that each one assumes that the other acts rationally. These are reasonable assumptions which were also made in McConnell and Schwartz (1986).

We also assume that the issuer tries to minimise the value of the instrument at all times and the holder tries to maximise it.

The instrument being priced depends on two variables: the price of the underlying stock and the rates in the market. Whether a correlation exists between them has some impact on the resulting price of the instrument. In our model, we will assume there is no correlation and the rationale is as follows: Interest rate that the issuer pays,  $R$ , can be modelled as:  $R = R_f + R_s$ , where  $R_f$  is the risk-free rate for the particular maturity and  $R_s$  is the spread specific to that issuer. In the US,  $R_f$  might be the yield of the Treasury bond for that maturity and  $R_s$  is the additional premi-



um the specific issuer pays over and above the treasury bond.

On one hand, we can say that the total return from any stock must be equal to at least the risk-free rate. If this were not the case, no investor would buy that stock. Thus, if treasury bills yield 6% per annum, you would not buy any stock whose expected total return was only 3%. We therefore observe a positive correlation between  $R_f$  and the total return on the stock.

On the other hand, when a particular issuer suffers a dramatic decline in stock prices, it will usually be associated with an increasing debt to equity ratio, possible credit downgrades and an increase in  $R_s$ . Hence we observe a negative correlation between  $R_s$  and the total return on the stock.

Hence  $R$  is a sum of two factors, one positively correlated with the stock and the other negatively. Thus we postulate that there is no correlation between  $R$  and the return on the stock.

The volatility of the one-year rate is assumed constant over the life of the bond. Although we recognise that this is a restrictive assumption, we felt that it was much less so than assuming a constant interest rate.

Like many other mathematical models, ours assumes that there are no arbitrage conditions or bid-ask spreads, that markets are frictionless, and that all information is available to everyone at the same time.

**Computational examples**

Moving back to the Abitibi-Price bond, which matures in nine years. The following benchmark bond yields are assumed for a Canadian

issuer with the same credit quality:

6 months.....	8.429%
1 year .....	8.751%
2 years.....	9.970%
3 years.....	10.210%
5 years.....	10.920%
7 years.....	11.360%
10 years.....	11.580%

The issuer can call the bond at par after two years provided the stock price has gone above \$18.75. Thereafter there is a declining barrier to the stock price. When the stock is above the barrier, the bond can be called. On the other hand, the holder can convert the bond at any time to 6.6666 stocks. We also assume that the expected total return from the stock is 7%, the dividend yield is 0%, the volatility of the stock price is 25% and the volatility of the yields is 10%.

We can now plot the computed bond price against various share prices (see figure above). On March 30, 1994 the stock was trading at \$17.375 and the bond was trading at \$130.00. The theoretical price of the bond was \$129.6161, which confirms that the model's price is close to the actual price.

This valuation method can be extended to other types of bonds, such as extendible and retractable ones and those with put options. ■

*Water Cheung is the head of derivatives and trading at Chase Manhattan's regional Asian currency desk in Hong Kong. Izzy Nelkei is the president of Super Computing Consulting, which develops software and trading systems for the financial community. The authors wish to acknowledge Dr Jack Mosevich of Harris Investment Management in Chicago*

**References**

Fabozzi, Frank J, *Bond Markets, Analysis and Strategies*, Prentice Hall, Englewood Cliffs, NJ, 2nd edition, 1993, page 376

McConnell, John J and Eduardo S Schwartz, 1986, *Lyon Taming*, *The Journal of Finance*, Vol XI, No. 3, July 1986

Kalotay, Andrew J, George O Williams and Frank J Fabozzi, 1993, *A model for valuing bonds and embedded options*, *Financial Analysts Journal*, May/June 1993, page 35

Hull, John C, 1993, *Options, futures and other derivative securities*, Prentice Hall, Englewood Cliffs, NJ, 2nd edition

Cox, J, S Ross and M Rubinstein, 1979, *Option pricing: a simplified approach*, *Journal of Financial Economics*, Vol. 7, October 1979, pages 229-264