

## **Merton's Model, Credit Risk, and Volatility Skews**

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## **Merton's Model, Credit Risk, and Volatility Skews**

### **Abstract**

In 1974 Robert Merton proposed a model for assessing the credit risk of a company by characterizing the company's equity as a call option on its assets. In this paper we propose a way the model's parameters can be estimated from the implied volatilities of options on the company's equity. We use data from the credit default swap market to compare our implementation of Merton's model with the traditional implementation approach.

## Merton's Model, Credit Risk, and Volatility Skews

The assessment of credit risk has always been important to banks and other financial institutions. Recently banks have devoted even more resources than usual to this task. This is because, under the proposals in Basel II, regulatory credit-risk capital may be determined using a bank's internal assessments of the probabilities that its counterparties will default.

One popular approach to assessing credit risk involves Merton's (1974) model. This model assumes that a company has a certain amount of zero-coupon debt that will become due at a future time  $T$ . The company defaults if the value of its assets is less than the promised debt repayment at time  $T$ . The equity of the company is a European call option on the assets of the company with maturity  $T$  and a strike price equal to the face value of the debt. The model can be used to estimate either the risk-neutral probability that the company will default or the credit spread on the debt.<sup>1</sup>

As inputs, Merton's model requires the current value of the company's assets, the volatility of the company's assets, the outstanding debt, and the debt maturity. One popular way of implementing Merton's model estimates the current value of the company's assets and the volatility of the assets from the market value of the company's equity and the equity's instantaneous volatility using an approach suggested by Jones *et al* (1984). A debt maturity date is chosen and debt payments are mapped into a single payment on the debt maturity date in some way.

In this paper we develop of new way of implementing Merton's model. This is based on using the implied volatilities of options issued by the company to estimate model parameters. Our approach is interesting both because it provides an alternative to Jones *et*

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<sup>1</sup> A number of authors such as Black and Cox (1976), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001) have developed interesting extensions of Merton's model, but none has emerged as clearly superior. See Eom *et al* (2002) which compares the performance of alternative models using bond spreads. Gemmill (2002) shows that Merton's model works well in the particular case where zero-coupon bonds are used for funding.

al and because it provides insights into the linkages between credit markets and options markets.

Under Merton's model an option on a company is a compound option on the assets of the company. Geske (1979), who provides a valuation formula for compound options, also shows that Merton's model is consistent with the type of volatility skew observed in equity markets.<sup>2</sup> In this paper we carry Geske's analysis one stage further to show that the credit spread in Merton's model can be calculated from the implied volatilities of two equity options. The options we choose are two-month at-the-money and out-of-the money put options.

To test our implementation of Merton's model and compare it with the more traditional implementation approach we use credit default swap (CDS) spread data. A CDS is a derivative that protects the buyer against default by a particular company. The CDS spread is the amount paid for protection and is a direct market-based measure of the company's credit risk. Most previous researchers have used bond data to test implementations of Merton's model. Using CDS spreads is an attractive alternative. Bond prices have the disadvantage that they are often indications rather than firm quotes. Also, the credit spread calculated from a bond price depends on the bond's liquidity and involves an assumption about the benchmark risk-free rate.

The rest of this paper is organized as follows. Section I develops the theory underlying our implementation of Merton's model. Section II describes the data we use. In Section III we compare the credit spreads implied by Merton's model with CDS spreads for both our implementation of Merton's model and the traditional implementation. In Section IV we present some results on the theoretical relationships between implied volatilities and credit spreads under Merton's model and test whether these relationships hold. In Section V we develop a relatively simple model, based on Merton's (1976) jump diffusion model, for relating credit spreads to implied volatilities and use it as a benchmark to test whether the more elaborate structure underlying our implementation of Merton (1974) provides a better explanation of observed credit spreads. Conclusions are in Section VI.

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<sup>2</sup> As the strike price of an equity option increases its volatility decreases. See Rubinstein (1994) and Jackwerth and Rubinstein (1996) for a discussion of this.

## I. Merton's Model

Both Merton (1974) and Black and Scholes (1973) propose a simple model of the firm that provides a way of relating credit risk to the capital structure of the firm. In this model the value of the firm's assets is assumed to obey a lognormal diffusion process with a constant volatility. The firm has issued two classes of securities: equity and debt. The equity receives no dividends. The debt is a pure discount bond where a payment of  $D$  is promised at time  $T$ .

If at time  $T$  the firm's asset value exceeds the promised payment,  $D$ , the lenders are paid the promised amount and the shareholders receive the residual asset value. If the asset value is less than the promised payment the firm defaults, the lenders receive a payment equal to the asset value, and the shareholders get nothing.

### A. Equity Value and the Probability of Default

Define  $E$  as the value of the firm's equity and  $A$  as the value of its assets. Let  $E_0$  and  $A_0$  be the values of  $E$  and  $A$  today and let  $E_T$  and  $A_T$  be their values at time  $T$ . In the Merton framework the payment to the shareholders at time  $T$ , is given by

$$E_T = \max[A_T - D, 0]$$

This shows that the equity is a call option on the assets of the firm with strike price equal to the promised debt payment. The current equity price is therefore

$$E_0 = A_0 N(d_1) - De^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(A_0 e^{rT} / D)}{\sigma_A \sqrt{T}} + 0.5 \sigma_A \sqrt{T}; \quad d_2 = d_1 - \sigma_A \sqrt{T}$$

$\sigma_A$  is the volatility of the asset value, and  $r$  is the risk-free rate of interest, both of which are assumed to be constant. Define  $D^* = De^{-rT}$  as the present value of the promised debt payment and let  $L = D^* / A_0$  be a measure of leverage. Using these definitions the equity value is

$$E_0 = A_0 [N(d_1) - LN(d_2)] \quad (1)$$

where

$$d_1 = \frac{-\ln(L)}{\sigma_A \sqrt{T}} + 0.5\sigma_A \sqrt{T}; \quad d_2 = d_1 - \sigma_A \sqrt{T}$$

As shown by Jones *et al* (1984), because the equity value is a function of the asset value we can use Ito's lemma to determine the instantaneous volatility of the equity from the asset volatility:<sup>3</sup>

$$E_0 \sigma_E = \frac{\partial E}{\partial A} A_0 \sigma_A$$

where  $\sigma_E$  is the instantaneous volatility of the company's equity at time zero. From equation (1), this leads to

$$\sigma_E = \frac{\sigma_A N(d_1)}{N(d_1) - LN(d_2)} \quad (2)$$

Equations (1) and (2) allow  $A_0$  and  $\sigma_A$  to be obtained from  $E_0$ ,  $\sigma_E$ ,  $L$  and  $T$ .<sup>4</sup> The risk-neutral probability,  $P$ , that the company will default by time  $T$  is the probability that shareholders will not exercise their call option to buy the assets of the company for  $D$  at time  $T$ . It is given by

$$P = N(-d_2) \quad (3)$$

This depends only on the leverage,  $L$ , the asset volatility,  $\sigma$ , and the time to repayment,  $T$ .

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<sup>3</sup> Jones et al (1984) actually use equations (1) and (2) in conjunction with some estimates of  $A$  and  $\sigma$ .

<sup>4</sup>The implementation of Merton's model, based on equations (1) and (2), has received considerable commercial attention in recent years. Moody's KMV uses it to estimate relative probabilities of default. CreditGrades (a venture supported by Deutsche Bank, Goldman Sachs, JP Morgan, and the RiskMetrics Group) uses it to estimate credit default swap spreads.

### B. Debt Value and the Implied Credit Spread of Risky Debt

Merton's model can be used to explain risky debt yields. Define  $B_0$  as the market price of the debt at time zero. The value of the assets at any time equals the total value of the two sources of financing so that

$$B_0 = A_0 - E_0$$

Using equation (1) this becomes

$$B_0 = A_0 [N(-d_1) + LN(d_2)] \quad (4)$$

The yield to maturity on the debt is defined implicitly by

$$B_0 = De^{-yT} = D^* e^{(r-y)T}$$

Substituting this into equation (4) and using  $A_0 = D^*/L$  gives the yield to maturity as

$$y = r - \ln [N(d_2) + N(-d_1)/L] / T$$

The credit spread implied by the Merton model is therefore<sup>5</sup>

$$s = y - r = -\ln [N(d_2) + N(-d_1)/L] / T \quad (5)$$

Like the expression for the risk-neutral probability of default in equation (3), the implied credit spread depends only on the leverage,  $L$ , the asset volatility,  $\sigma_A$ , and the time to repayment,  $T$ .

### C. Equity Volatility and Volatility Skews

One point about Merton's model that has not received much exploration is the role it plays in explaining equity implied volatilities and the volatility skews that are observed in the equity options market. Within the framework of the Merton model, an option on the firm's equity that expires before the debt matures is a compound option, an option on a European call option. We can therefore use the model proposed by Geske (1979) to

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<sup>5</sup> The relationship between credit spreads and default risk is discussed by Duffie and Singleton (1999), Litterman and Iben (1991), and Rodriguez (1988) among others.

evaluate it. Using the notation developed above, the value at time zero of a put with strike price  $K$  and expiry time  $\tau < T$  on the equity is

$$P = De^{-rT} M\left(-a_2, d_2; -\sqrt{\tau/T}\right) - A_0 M\left(-a_1, d_1; -\sqrt{\tau/T}\right) + Ke^{-r\tau} N(-a_2) \quad (6)$$

where

$$a_1 = \frac{\ln\left(A_0 / A_\tau^* e^{-r\tau}\right)}{\sigma_A \sqrt{\tau}} + 0.5\sigma_A \sqrt{\tau}; \quad a_2 = a_1 - 0.5\sigma_A \sqrt{\tau}$$

$M$  is the cumulative bivariate normal distribution function, and  $A_\tau^*$  is the critical asset value at time  $\tau$ , the value for which the equity value at that time equals  $K$ . That is  $A_\tau^*$  is the asset value below which the put on the equity will be exercised.

Define  $\nu$  as the implied volatility of the put at time zero based on the Black-Scholes model. Also define parameters  $\alpha$  and  $\kappa$  by

$$A_\tau^* = \alpha A_0 e^{r\tau}$$

$$K = \kappa E_0 e^{r\tau}$$

The parameter  $\alpha$  is the ratio of the critical asset price to the forward asset price (with both being observed at time zero). We will refer to it as the implied strike level. The parameter  $\kappa$  is the ratio of the option strike price to the forward equity price (observed at time zero). We will refer to it as the option's moneyness.

Implied volatilities are the volatilities which, when substituted into the Black-Scholes model, give the market price. If we assume that market prices are given by Merton's model, the implied volatility of an option can be determined by solving:

$$\begin{aligned} D^* M\left(-a_2, d_2; -\sqrt{\tau/T}\right) - A_0 M\left(-a_1, d_1; -\sqrt{\tau/T}\right) + \kappa E_0 N(-a_2) \\ = \kappa E_0 N(-d_2^*) - E_0 N(-d_1^*) \end{aligned} \quad (7)$$

where

$$d_1^* = \frac{-\ln(\kappa)}{v\sqrt{\tau}} + 0.5v\sqrt{\tau}; \quad d_2^* = d_1^* - v\sqrt{\tau}$$

$$a_1 = \frac{-\ln(\alpha)}{\sigma_A\sqrt{\tau}} + 0.5\sigma_A\sqrt{\tau}; \quad a_2 = a_1 - \sigma_A\sqrt{\tau}$$

Substituting equation (1) into equation (7) results in

$$LM(-a_2, d_2; -\sqrt{\tau/T}) - M(-a_1, d_1; -\sqrt{\tau/T}) + \kappa N(-a_2)[N(d_1) - LN(d_2)]$$

$$= [\kappa N(-d_2^*) - N(d_1^*)] [N(d_1) - LN(d_2)] \quad (8)$$

A variation of equation (1) can be used to determine the implied strike level,  $\alpha$ :

$$\kappa E_0 e^{r\tau} = A_\tau^* [N(d_{1,\tau}) - (L/\alpha)N(d_{2,\tau})]$$

so that

$$\kappa = \frac{\alpha N(d_{1,\tau}) - LN(d_{2,\tau})}{N(d_1) - LN(d_2)} \quad (9)$$

where

$$d_{1,\tau} = \frac{-\ln(L/\alpha)}{\sigma_A\sqrt{T-\tau}} + 0.5\sigma_A\sqrt{T-\tau}; \quad d_{2,\tau} = d_{1,\tau} - \sigma_A\sqrt{T-\tau}$$

Equations (8) and (9) define an implicit relationship between the implied volatility of an option and the moneyness,  $\kappa$ , for a set of model parameter values  $L$ ,  $\sigma_A$ , and  $T$ , and the option maturity,  $\tau$ . For different values of  $\kappa$  different implied volatilities,  $v$ , will result leading to a volatility skew. For all values of the model parameters, the implied volatilities are of the form observed in practice where an increase in the strike price leads to a reduction in the implied volatility.

#### *D. An Alternative Implementation of Merton's Model*

Section I.C suggests a new way of implementing Merton's model using two implied volatilities. With one implied volatility we can solve equations (8) and (9) for a particular value of  $T$  to obtain a relationship between the leverage ratio,  $L$ , and the asset volatility,  $\sigma_A$ . With two implied volatilities we have two such relationships that can be solved for  $L$

and  $\sigma_A$ . The credit spread for a zero-coupon bond maturing at time  $T$  can then be calculated using equation (5).

This implementation approach allows credit spreads to be estimated directly from implied volatility data. It is a potentially attractive alternative to the traditional implementation based on equations (1) and (2) because it avoids the need to estimate the instantaneous equity volatility and the need to map the company's liability structure (some of which may be off balance sheet) into a single zero-coupon bond.

In the sections that follow we will compare the results from our implementation of Merton's model with the traditional implementation. As a benchmark we will also examine the performance of a simpler model where a Poisson process generates defaults.

## **II. Data**

Our empirical tests are based on credit default swap data, implied volatility data, equity price data, and balance sheet data for companies between January and December 2002.

### *A. Credit Default Swap Data*

A credit default swap or CDS provides insurance against a default by a particular company or sovereign entity. The company is known as the reference entity and a default by the company is known as a credit event. The buyer of the insurance makes periodic payments to the seller and in return obtains the right to sell a bond issued by the reference entity for its face value if a credit event occurs. The amount of the payments made per year by the buyer is known as the CDS spread.<sup>6,7</sup> The credit default swap market has

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<sup>6</sup> In a standard contract, payments by the buyer are made quarterly or semiannually in arrears. If the reference entity defaults, there is a final accrual payment and payments then stop. Contracts are sometimes settled in cash rather than by the delivery of bonds. In this case there is a calculation agent who has the responsibility of determining the market price,  $x$ , of a bond issued by the reference entity a specified number of days after the credit event. The payment by the seller is then is  $100-x$  per \$100 of principal.

<sup>7</sup> The CDS spread is very close to the credit spread observed in the corporate bond market when the credit spread is measured relative to the swap rate. For more information on the relation between CDS spreads and bond credit spreads see Blanco et al (2003), Longstaff et al (2003) or Hull et al (2003).

grown rapidly since the International Swaps and Derivatives Association produced its first version of a standardized contract in 1998.<sup>8</sup>

GFI, a broker specializing in the trading of credit derivatives provided us with CDS quotes for the period January to December 2002. The data contains in excess of 120,000 individual CDS quotes. Each quote contains the following information:

- The date on which the quote was made,
- The name of the reference entity,
- The life of the CDS,
- Whether the quote is a Bid (wanting to buy protection) or an Offer (wanting to sell protection), and
- The CDS spread quote in basis points.

Each quote is a firm quote for a minimum notional of 10 million USD. The reference entity may be a corporation such as Blockbuster Inc., a sovereign such as Japan, or a quasi-sovereign such as the Federal Home Loan Mortgage Corporation. During the period we are considering CDS quotes are provided on 1,597 named entities: 1,500 corporations, 60 sovereigns and 37 quasi-sovereigns. Of the reference entities 796 are North American, 451 are European, and 330 are Asian and Australian. The remaining reference entities are African or South American.

We used only five-year quotes in our analysis. (Approximately 85% of the quotes in our data set were for five year CDS.) When there were both five-year bid and offer quotes on a reference entity in a day, we calculated what we will refer to as an "observation" on the five-year CDS spread as the average of the maximum bid and minimum offer.<sup>9</sup> As shown by Duffie (1999) and Hull and White (2000), the five-year credit default swap spread is in theory very close to the credit spread of the yield on a five-year par yield bond issued by the reference entity over five-year par yield risk-free rate.

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<sup>8</sup> A more complete description of credit default swaps and the CDS market can be found in, for example, Duffie (1999).

<sup>9</sup> When there was a trade the bid equals the offer.

### *B. Implied Volatility Data*

From the list of entities covered by the CDS data, we chose optionable US equities with current stock prices over \$20. This resulted in a list of 325 companies. The Bloomberg system, which has been archiving the implied volatility of options on US equities since the beginning of 2002, was polled to determine which of the names had implied volatility data available. This reduced the sample size to 319 firms. For this sample, the implied volatility for the two-month 50-delta put and the 25-delta put were downloaded for every trading day in the year 2002. This produced 61,544 observations.

In the Bloomberg system a two-month option is defined as an option with a maturity between one and two months. In most circumstances the exact maturity date of the option can be calculated from knowledge of the maturity dates of options traded on the Chicago Board Option Exchange. The two-month 25-delta put implied volatility is an estimate of the implied volatility of a two-month put option that has a delta of  $-0.25$ . This is calculated by interpolating between the implied volatility of the two two-month options whose deltas are closest to  $-0.25$ , one greater than  $-0.25$  and the other less than  $-0.25$ . The implied volatility for a two-month 50-delta put is calculated similarly. We chose the 50-delta and 25-delta implied volatilities because they are usually calculated from relatively liquid options.

### *C. Data for Traditional Implementation*

For the traditional approach to implementing Merton's model we downloaded daily closing stock prices and quarterly balance sheet information from Bloomberg for the period between January 1, 2001 and December 31, 2002. We used the reported quarterly total liabilities divided by the quarterly reported outstanding number of shares as our measure of the debt claim,  $D$ . On any particular day the current value of the debt claim was set to the most recently reported value. The equity price and the instantaneous equity volatility used in equations (1) and (2) were the daily closing price and a historic volatility estimated using the most recent 40 returns. The choice of 40 business days to estimate volatility is a trade-off. Using a longer (shorter) period to estimate volatility

results in a more (less) accurate but less (more) timely estimates. The total number of observations was 63,627.

#### *D. Merged Data*

We merged the volatility data and the data used in the traditional implementation with the CDS data. This combined data set was then filtered to eliminate cases in which the volatility exceeded 90% (65 cases) and firms for which there were 10 or fewer implied volatility observations or CDS quotes. This resulted in a pool of 127 firms, each with between 11 and 178 days of data for a total of 6,220 firm days of data. In other words we ended up with 6,220 cases where for a particular company on a particular day we had a) a CDS spread observation, b) a two-month 50-delta put implied volatility, c) a two-month 25-delta put implied volatility, d) an equity value, e) a debt claim, and f) an historic equity volatility estimate. This is the data that was used for the analysis described in the following sections.

### **III. Empirical Tests**

In this section we test whether five-year credit spreads implied from our implementation of Merton's model and the traditional implementation are consistent with the observed five-year CDS spreads. For the traditional implementation we estimated the equity value, historical equity volatility and the outstanding debt in the way described in Section II.C. Equations (1) and (2) were used to compute the asset value and asset volatility and equation (5) was then used to compute the credit spread. For our implementation we used the 50- and 25-delta implied put volatilities in equations (8) and (9) to imply the leverage ratio and the asset volatility.<sup>10</sup> As in the case of the traditional implementation, equation (5) is then used to compute the credit spread. In the balance of the discussion, we shall refer to our implementation of Merton's as the ImpVol implementation and the traditional implementation as the Trad implementation.

There are a number of reasons why we should expect differences between the credit spreads implied from Merton's model and observed CDS spreads. Merton's model is not

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<sup>10</sup> This involves determining the value of  $\kappa$  that produces options with deltas of 0.50 and 0.25.

a perfect representation of reality because companies do not usually issue only zero coupon debt and because a number of factors besides the value of their assets are liable to influence a company's decision to default on its obligations. Also, CDS spreads are liable to be slightly different from bond yield spreads for the reasons listed in Hull et al (2002). Finally, the credit spread backed out from Merton's model is the spread between the yields on zero coupon bonds while a CDS credit spread is (at least approximately) the spread between the yields on par yield bonds.

Table I shows the results of regressing CDS spreads against the spreads implied from Merton's model using the two implementation approaches. Given the nature of the model generating the implied spreads it is unlikely that errors in the implied spreads are normally distributed. As a statistical test the regression is therefore of doubtful validity. However it does provide a first attempt at describing the relationship (if any) between the implied and observed spreads.

The results in Table I reveal a positive relationship between the observed CDS spreads and the implied spreads that is roughly similar for both implementations. The mean CDS observed spread about 95 basis points higher than the mean implied spread for both models. The  $R^2$  of the regressions indicate that the ImpVol implementation provides a better fit to the observed data than the Trad implementation.

It is possible that there are factors other than those suggested by Merton's model that affect CDS spreads. Figure 1 shows a scatter diagram of the CDS spread versus the implied spread from the ImpVol model for Merrill Lynch, Dow Chemicals, and Bowater. Figure 2 shows the same for the Trad model. These figures suggest that the relation between the CDS spread and implied spread may be different for different firms. It is also possible that macroeconomic variables cause the relationship between the CDS spread and implied spread to change through time. To explore these possibilities we carried out a separate regression for each firm and a separate regressions for each day.

The results are shown in Table II. For both the firm-by-firm regressions and the day-by-day regressions we report mean and median values for the constant, slope,  $R^2$ , and number of observations. Any firm for which there were less than 30 observations was not included in the firm-by-firm regressions. Any day for which there were less than 30

observations was not included in the day-by-day regression. This resulted in 86 firm-by-firm regressions and 90 day-by-day regressions. Table II shows that on average the implied spreads fit the CDS observed spreads much better when considered on a firm-by-firm or day-by-day basis than when fitting the entire sample. This is not surprising since the number of degrees of freedom is much larger in the firm-by-firm or day-by-day analysis. On the basis of the regression  $R^2$ , the results also show that the implied spreads match CDS spreads better on a firm-by-firm basis than when looking across firms on a particular day. What this means is that the models seem to work better at explaining how the observed credit spreads for a firm change over time than they do at discriminating between different firms at a single time. The two implementations are comparable when applied on a firm-by-firm basis but the ImpVol implementation appears to provide a better fit than the Trad implementation when applied cross-sectionally.

#### A. Rank Correlations

The results so far presented indicate that both implementations of Merton's model are consistent with the data in the sense that there is a positive relationship between the model predictions and the observed data. There is also some evidence that the relation between the implied credit spread and observed credit spread is non-linear.

To address the apparent non-linearity we could test alternative non-linear models. However, the nature of the non-linearity is not known and may differ from firm to firm. A general approach to fitting data subject to an unknown non-linear relationship is to linearize the data by translating observations to ranks. Formally if

$$y = f(x)$$

for some monotonic increasing function,  $f$ , and we have a set of observations  $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$  then

$$r(y_i | \{y_1, y_2, \dots, y_n\}) = r(x_i | \{x_1, x_2, \dots, x_n\})$$

where  $r(a | \{b\})$  is the rank of  $a$  within the set  $b$ . This linearization works perfectly in the deterministic case described here, but difficulties arise when variables are observed with error. However, there is a well developed literature on rank correlation which will allow a

more formal and robust distribution-free test of which version of the model is more consistent with the data.<sup>11</sup>

There are two measures of rank order correlation in the literature: Kendall's and Spearman's. To explain how they are calculated suppose that we have  $n$  observations on two variables. (In our application the variables are five-year implied credit spread and five-year observed CDS spread). As a first step we calculate the rank of each observation on each variable.

For the Kendall rank order correlation measure,  $r_k$ , we look at the  $n(n-1)/2$  alternative pairs of observations. If the rankings of variables for a particular pair of observations are in the same order we score +1 and if they are in inverse order we score -1. For example, if for a particular pair of observations, the rankings of the first variable were 5 and 10, respectively, and the rankings of the second variable were 6 and 8, respectively, we would score +1. If the rankings of the first variable were 5 and 10, respectively, and the rankings of the second variable were 8 and 6, respectively, we would score -1. The rank order correlation is the sum of the scores for all pairs divided by  $n(n-1)/2$ .

For the Spearman's rank order correlation,  $r_s$ , we calculate for each observation  $i$  the difference,  $d_i$ , between the rank order of the first variable and the rank order of the second variable. The correlation measure is

$$r_s = 1 - \frac{6 \sum_{i=1}^n d_i^2}{n^3 - n}$$

Kendall and Gibbons (1990) provides a great deal of information on the properties of the two correlation measures. The properties we will use and the statistical tests they give rise to are outlined in Appendix A.

The formal statistical tests based on the rank order correlation between implied spreads and observed CDS spreads are reported in Table III. The first panel in Table III shows results from pooling all the data. The middle panel carries out a firm-by-firm analysis. Rank order correlations were calculated for every company that had 30 or more

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<sup>11</sup> In the context of our tests it is interesting to note that proponents of the commercial use of Merton's model claim that, although estimated credit spreads are not accurate, the model does at least rank the credit quality of companies well. See, for example, Crosbie (1998).

observations as in Table II. The final panel in Table III tests whether Merton's model can be used to rank the relative credit quality of different firms at a point in time. Rank correlations were computed for every day on which data was available for 30 or more firms as in Table II.

The broad conclusions from the results in Table III are similar to those from the results in Tables I and II. The  $z$ -statistics show that we can reject the null hypothesis that the rank order correlations are zero with a very high degree of confidence in all cases for both implementations of Merton's model. The rank order correlations within firms across time are always higher than those measured within a day across firms.<sup>12</sup> This indicates that both implementations do better at tracking a single firm over time than they do at distinguishing between firms at a point in time.

The correlations from the ImpVol implementation of Merton's model are always higher than for the Trad implementation. In the case of the firm-by-firm analysis they are significantly higher at the 1% level. This is in contrast to the results in Table II where the  $R^2$  for Trad is slightly higher than the  $R^2$  for ImpVol in the firm-by-firm case. A close inspection revealed the  $R^2$  for the firm-by-case case in Table II was greatly influenced by a few outliers. Outliers have far less effect on rank order correlations.

There are a number of possible reasons why the ImpVol implementation ranks credit spreads better than the Trad implementation. There is the noise in estimates of the historic volatility for the Trad implementation and the Trad implementation requires the assumption that the historic volatility is the same as the instantaneous volatility. Also we had only limited information on the company's capital structure for the Trad implementation.

### *B. Impact of Debt Maturity*

As the debt maturity date in the ImpVol implementation of Merton's model changes the implied credit spread changes, but there is little effect on the ranking of implied credit spreads. This is illustrated in Table IV, which shows the rank order correlation between

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<sup>12</sup> Although the calculations are not shown, in every case the within firm rank order correlation is significantly larger than the corresponding within day rank order correlation at the 1% level.

the implied credit spreads when  $T$  equals 1, 2, 5, and 10. We obtained similar results for the probability of default rankings and for the day-by-day and company-by-company analysis. For financial institutions that are interested only in ranking the creditworthiness of counterparties this result may add to the attraction of the ImpVol implementation of Merton's model.

#### **IV. Properties of Merton's Model**

In this section we show that Merton's model makes certain predictions about the nature of the relationship between implied volatilities and credit spreads and test whether the predictions are supported by the data. We define ATMVOL for maturity  $\tau$  as the at-the-money implied volatility for an option with a delta of 0.50. We define SKEW for maturity  $\tau$  as the implied volatility for an option with maturity  $\tau$  and a delta of 0.25 minus the implied volatility for an option with maturity  $\tau$  and a delta of 0.50.

Our implementation of Merton's model can be used to relate  $T$ -year credit spreads to ATMVOL and SKEW for particular values of  $\tau$  and  $T$ . Figure 3 shows the theoretical relationship between credit spread and ATMVOL for different values of SKEW when  $T = 5$  and  $\tau = 0.1667$ . The figure shows that there is a pronounced positive relationship with positive convexity. Similar results are obtained for other values of  $T$  and  $\tau$ . Figure 4 shows the relationship between credit spread and SKEW for different values of ATMVOL when  $T = 5$  and  $\tau = 0.1667$ . There is very little relationship for low values of ATMVOL (volatility less than 50%) but a strongly positive relationship for higher levels of ATMVOL.

To test whether these properties of the model are supported by the data we performed a linear regression of observed CDS spread (CREDSR) against ATMVOL and SKEW for our data:

$$\text{CREDSR} = a + b \times \text{ATMVOL} + c \times \text{SKEW}$$

ATMVOL and SKEW are quite highly correlated. To address this co-linearity issue the regressions were done twice, once orthogonalizing SKEW with respect to ATMVOL and once orthogonalizing ATMVOL with respect to SKEW. To explore both the convexity of

the relation between CREDSR and ATM and the increasing slope of the relation between CREDSR and SKEW for higher ATM volatilities, we partitioned the sample into subsets based on the ATM volatility<sup>13</sup>. The results are shown in Table V.

#### *A. Relation between Credit Spread and ATM Volatility*

When SKEW is orthogonalized with respect to ATMVOL, Table V shows that as predicted there is a significantly positive slope for the ATMVOL for the full sample. When the sample is split into two roughly equal subsets based on ATMVOL we find that the coefficient of the ATMVOL is consistently significantly positive. Further the coefficient of the ATMVOL is higher in the high volatility subset than it is in the low volatility subset. This is consistent with the convex relationship predict by the model. The increase in slope is statistically significant.

When ATMVOL is orthogonalized with respect to SKEW some of the impact of the positive relation between CREDSR and ATMVOL is liable to be captured by the SKEW coefficient. However, it is reassuring that the coefficient ATMVOL is still always significantly positive at the 1% level. Furthermore it is significantly higher for the high volatility subset than for the low volatility subset. Table V therefore provides strong evidence for a convex positive relation between CREDSR and ATMVOL.

#### *B. Relation between Credit Spread and Skew*

When ATMVOL is orthogonalized with respect to SKEW the relation between CREDSR and SKEW is always significantly positive at the 1% level. When the sample is split into two groups, the coefficient of SKEW is significantly higher in the high volatility group than in the low volatility group and the difference is significant.

When SKEW is orthogonalized with respect to ATMVOL, the coefficient of SKEW is not significant overall. It is not significant for the lower volatility subset and is significant at the 5% level for the high volatility subset. This is consistent with the model, although less strongly so than the results in Section IV.A.

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<sup>13</sup> The partitioning was based on the average ATM volatility for each firm. As a result each firm appears in only one subset.

A general observation is that the volatility skews observed in practice are much higher than those than could reasonably be predicted by Merton's model. The reason may be what Rubinstein (1994) has referred to as crash-o-phobia.

## VI. Does the Merton Model Add Explanatory Power?

As shown by equations (8) and (9) Merton's model provides a fairly complex relationship between implied volatilities and credit spreads. In this section we develop as a benchmark a much simpler model, with far less structure, for relating credit spreads to implied volatilities. We test whether Merton's (1974) model outperforms this benchmark.

In Merton's (1976) jump diffusion model, the stock price obeys the process:

$$\frac{dS}{S} = (\mu - \lambda k) dt + \sigma_E dz + dq$$

where  $dq$  is a Poisson process with intensity  $\lambda$  and jump size  $k$ . The probability that a jump occurs in some small time interval,  $dt$ , is  $\lambda dt$ . In the event of a jump the change in the stock price is  $dS = kS$ . The expected return on the stock is

$$E(dS/S)/dt = (\mu - \lambda k) + \lambda k = \mu$$

We consider the particular case of this model where  $k = -1$ . In this case jumps always lead to a zero stock price. We assume that a zero stock price coincides with a default. Defaults are therefore generated by a Poisson process with intensity  $\lambda$ .<sup>14</sup>

Consider a put option with strike price  $K$  and time to maturity  $\tau$ . As before we define a moneyness variable  $\kappa$  by

$$K = \kappa S_0 e^{r\tau}$$

where  $S_0$  is the today's stock price. Merton showed that if the jumps are not priced<sup>15</sup> the option's price is

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<sup>14</sup> This is similar to reduced form models such as those proposed by Duffie and Singleton (1999)

<sup>15</sup> The variable,  $\lambda$ , is the intensity under the risk-neutral measure.

$$S_0 \left[ \kappa e^{-\lambda\tau} N(-d_2) - N(-d_1) + \kappa(1 - e^{-\lambda\tau}) \right] \quad (10)$$

where

$$d_1 = \frac{\lambda - \ln \kappa}{\sigma_E \sqrt{\tau}} + 0.5\sigma_E \sqrt{\tau}; \quad d_2 = d_1 - \sigma_E \sqrt{\tau}$$

Equating this option price to the Black-Scholes price defines the implied volatility  $v$ :

$$\kappa e^{-\lambda\tau} N(-d_2) - N(-d_1) + \kappa(1 - e^{-\lambda\tau}) = \kappa N(-d_2^*) - N(-d_1^*) \quad (11)$$

where as before

$$d_1^* = \frac{-\ln(\kappa)}{v\sqrt{\tau}} + 0.5v\sqrt{\tau}; \quad d_2^* = d_1^* - v\sqrt{\tau}$$

If a volatility,  $\sigma_E$ , and a default intensity,  $\lambda$ , are chosen and equation (11) is solved to determine the Black-Scholes implied volatility for various values of  $\kappa$ , a volatility skew results. Similarly to the case of the Merton model discussed in Section I.C, the lower the value of  $\kappa$ , the higher is the implied volatility,  $v$ .<sup>16</sup>

Merton's (1976) model can also be used to determine the price of a zero-coupon bond maturing at time  $T$ , issued by the firm. If no default has occurred the bond is assumed to pay \$1 at maturity and if a default has occurred at or before maturity the bondholder is assumed to recover  $R \leq \$1$  at maturity.<sup>17</sup> Conditional on the intensity of the Poisson process, the risk-neutral probability that no default has occurred before maturity,  $\pi$ , is  $\exp[-\lambda T]$  and the probability that a default has occurred is  $1 - \pi$ . The bond price is the present value of the risk-neutral expected value discounted back to the present using the risk free rate of interest.

$$[\pi + (1 - \pi)R]e^{-rT} = e^{-yT}$$

where  $y$  is the yield on the zero-coupon bond. The credit spread on the debt is then

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<sup>16</sup> A version of the jump-diffusion model in which the diffusion part of the process obeys a constant elasticity of variance process was also tested. The results from the CEV version were not materially different from the results from the version described in the text and are not reported.

<sup>17</sup> This is equivalent to the assumption that the claim in the event of default is proportional to the default risk free value of the debt.

$$y - r = -\ln[\pi + (1 - \pi)R]/T$$

or

$$-\ln[e^{-\lambda T} + (1 - e^{-\lambda T})R]/T \quad (12)$$

In the event that the recovery rate,  $R$ , is zero the credit spread equals the default intensity,  $\lambda$ , for all maturities.

Just as equations (8), (9) and (5) allowed us to imply credit spreads from option volatilities under the Merton (1974) model, equations (11) and (12) allow us to do so under the Merton (1976) model. Option implied volatilities can be used to infer the default intensity and the stock volatility and these parameters can be used to imply a credit spread. As in Section III this can be compared with the contemporaneous CDS spread. In doing the analysis it is necessary to assume a time to debt maturity,  $T$ , and a recovery rate,  $R$ . The magnitude of the resulting implied credit spread is sensitive to these assumptions but the relative ranking of outcomes is not.

These results allow us to provide an interesting test of the value of the structural model underlying Merton (1974). Our null hypothesis is that Merton (1976) ranks the credit quality of companies as well as Merton (1974). Table VI compares the performance of Merton's (1976) model with Merton's (1974) model. The 1976 model has statistically significant explanatory power but in all cases the Merton (1974) model provides significantly better predictions of default probabilities and credit spreads at the 1% level.

## VI. Conclusion

The traditional approach to implementing Merton's model involves estimating the instantaneous equity volatility and the debt outstanding by a particular future time. We have presented an alternative implementation where the inputs to the model are much simpler. All that is required to imply a credit spread is two implied volatilities.

Our proposed implementation of Merton's model outperforms a simple version of the traditional implementation of the model. It is reassuring that it also outperforms an

alternative way of deriving credit spreads from implied volatilities that is based on a model with less structure.

Two predictions made by Merton's are

- a) There should be a positive relationship with positive convexity between credit spreads and at-the-money volatilities.
- b) There should be a positive relationship between credit spreads and volatility skews when the at-the-money volatility is high.

The first of the predictions is strongly supported by the data. The second is also supported by the data, but somewhat less strongly than the first.

## Appendix. Statistical Properties of Rank Order Correlation

Kendall and Gibbons (1990) provides a great deal of information on the statistical properties of the Kendall's and Spearman's rank correlation measures. For  $n > 10$ , the probability distribution of Kendall's rank order correlation,  $r_k$ , conditional on no rank order correlation between the variables, is approximately normal with a mean of zero and a variance of  $[2(2n+5)]/[9n(n-1)]$ . The  $z$ -statistic for testing the null hypothesis that  $r_k$  is zero is therefore

$$\frac{3r_k\sqrt{n(n-1)}}{\sqrt{2(2n+5)}}$$

For  $n > 30$  the probability distribution of Spearman's rank order correlation,  $r_s$ , conditional on no rank order correlation is approximately normal with a mean of zero and a variance of  $1/(n-1)$ . The  $z$ -statistic for testing the null hypothesis that  $r_s$  is zero is therefore  $r_s\sqrt{n-1}$ .

When the rank order correlation is non-zero, Kendall and Gibbons show that the standard deviation of the estimate of  $r_k$  depends on the true value of  $r_k$  and other unknown quantities concerned with the arrangement of the ranks in the parent population. The same is true of  $r_s$ . The estimated value of  $r_k$  can be assumed to be drawn from a normal distribution with a mean of  $\rho_k$  and a variance of at most  $2(1-\rho_k^2)/n$  where  $\rho_k$  is the true Kendall rank order correlation. The estimated value of  $r_s$  can be assumed to be drawn from a normal distribution with a mean of  $\rho_s$  and a variance of at most  $3(1-\rho_s^2)/n$  where  $\rho_s$  is the true Spearman's rank order correlation. In practice  $\rho_k$  and  $\rho_s$  are set equal to the estimates,  $r_k$  and  $r_s$ , in these formulas.

These results enable us to construct a conservative test of whether there is a significant difference between two rank order correlations. For example, suppose we observe a Spearman rank order correlation of  $r_{s,1}$  from a sample of  $n_1$  and a Spearman rank order correlation of  $r_{s,2}$  from a sample of  $n_2$ . The  $z$ -statistic for testing whether they are significantly different is

$$\frac{r_{s,1} - r_{s,2}}{\sqrt{3(1-r_{s,1}^2)/n_1 + 3(1-r_{s,2}^2)/n_2}} \quad (\text{A1})$$

Suppose that  $r_{k,j}$  and  $r_{s,j}$  are the Kendall's rank order correlation and Spearman's rank order correlation for company  $j$ , that  $n_j$  is the number of observations for company  $j$ , and that there are  $N$  companies. Under the null hypothesis that there is no correlation between the two variables each of the  $r_{k,j}$  is normally distributed with mean zero and variance

$$V_j = 2(2n_j + 5)/(9n_j(n_j - 1))$$

The mean value of the  $r_{k,j}$  is then normally distributed with mean zero and variance

$$\frac{1}{N^2} \sum_{j=1}^N V_j$$

and the  $z$ -statistic for testing whether the mean is significantly different from zero is

$$z_k = \sum_{j=1}^N r_{k,j} / \sqrt{\sum_{j=1}^N 2(2n_j + 5)/(9n_j(n_j - 1))}$$

An upper bound for the standard error of each of the  $r_{k,j}$  is  $2(1-r_{k,j}^2)/n_j$  and the standard error of the mean value of the  $r_{k,j}$  is

$$\frac{1}{N} \sqrt{\sum_{j=1}^N 2(1-r_{k,j}^2)/n_j}$$

Analogously, the  $z$ -statistic for the mean value of the  $r_{s,j}$  is

$$z_s = \sum_{j=1}^N r_{s,j} / \sqrt{\sum_{j=1}^N \frac{1}{n_j - 1}}$$

and an upper bound for the standard error of the mean value of the  $r_{s,j}$  is

$$\frac{1}{N} \sqrt{\sum_{j=1}^N 3(1-r_{s,j}^2)/n_j}$$

The expressions for the  $z$ -statistics and standard error for daily means in the day-day analysis are similar to those in the company-by-company analysis. In this case  $j$  counts days rather than companies and  $N$  is the number of days for which we are able to calculate rank order correlations.

## REFERENCES

Black, Fischer, and John Cox, 1976, Valuing Corporate Securities: Some Effects of Bond Indenture Provisions, *Journal of Finance*, 31, 351-367.

Black, Fischer and Myron. Scholes, 1973, The Pricing of Options and Corporate Liabilities, *Journal of Political Economy*, 81, 637-59.

Blanco, Roberto S., Simon Brennan, and Ian W. Marsh, 2003 An empirical analysis of the dynamic relationship between investment grade bonds and credit default swaps, Bank of England Working paper.

Collin-Dufresne, Pierre, and Robert Goldstein, 2001, Do Credit Spreads Reflect Stationary Leverage Ratios, *Journal of Finance*, 52, 1929-1957.

Crosbie, Peter, 1998, Modeling Default Risk, in S. Das (ed.) *Credit Derivatives: Trading and Management of Default Risk*, John Wiley and Sons, Singapore.

Duffie, Darrell, 1999, Credit Swap Valuation, *Financial Analysts Journal*, 55, 73-89.

Duffie, Darrell., and Kenneth Singleton, 1999, Modeling Term Structures of Defaultable Bonds, *Review of Financial Studies*, 12, 687-720.

Eom, Young H., Jean Helwege, and Jing-zhi Huang, 2000, Structural Models of Corporate Bond Pricing: An Empirical Analysis, Working Paper no 2000-16, Pennsylvania State University.

Gemmill, Gordon, 2002, Testing Merton's Model for Credit Spreads on Zero-coupon Bonds, Working paper, City University Business School, London.

Geske, Robert., 1977, The Valuation of Corporate Liabilities as Compound Options, *Journal of Financial and Quantitative Analysis*, 5, 541-552

Geske, Robert., 1979, The Valuation of Compound Options, *Journal of Financial Economics*, 7, 63-82.

Hull, John, Mirela Predescu, and Alan White, 2002, The Relationship between Credit Default Swap Spreads, Bond Yields, and Credit Ratings Announcements, Working paper, University of Toronto.

Hull, John, and Alan White, 2000, Valuing Credit Default Swaps I: No Counterparty Default Risk, *Journal of Derivatives*, 8, 29-40.

Jackwerth, Jens C., and Mark Rubinstein, 1996, Recovering Probabilities from Option Prices, *Journal of Finance*, 51, 1611-31.

Jones, E. Philip, Scott P. Mason, and Eric Rosenfeld, 1984, Contingent Claims Analysis of Corporate Capital Structure: An Empirical Investigation, *Journal of Finance*, 39, 611-25.

Kendall, Maurice. and Jean D. Gibbons, 1990, *Rank Correlation Methods*, (5<sup>th</sup> ed.), Edward Arnold, London.

Leland, Hayne E. and Klaus B. Toft, 1996, Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads, *Journal of Finance*, 51, 987-1019.

Litterman, Robert and Thomas Iben, 1991, Corporate Bond Valuation and the Term Structure of Credit Spreads, *Journal of Portfolio Management*, 17, 52-64.

Longstaff, Francis A. and Eduardo Schwartz, 1995, A Simple Approach to Valuing Risky Fixed and Floating Debt, *Journal of Finance*, 50, 789-819.

Longstaff, Francis A., Sanjay Mithal, and Eric Neis, 2003, The credit-default swap market: Is Credit Protection Priced Correctly?, Working paper.

Merton, Robert C., 1974, On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance*, 29, 449-70.

Merton, Robert C., 1976, Option Pricing When Underlying Stock Returns are Discontinuous, *Journal of Financial Economics*, 3, 125-44.

Rodriguez, Ricardo J., 1988, Default Risk, Yield Spreads, and Time to Maturity, *Journal of Financial and Quantitative Analysis*, 23, 111-117.

Rubinstein, Mark, 1994, Implied Binomial Trees, *Journal of Finance*, 49, 771-818.

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**Table I**

**Regression of CDS spreads against Implied Credit Spreads.**

The credit spreads are implied from ImpVol implementation and Trad implementation of Merton's (1974) model. The ImpVol implementation is the implementation we propose in equation (8) and (9). The Trad implementation is the traditional implementation of Merton's model in equations (1) and (2). Standard errors are shown in parentheses.

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<u>Implementation</u>	<u>Constant</u>	<u>Slope</u>	<u>R<sup>2</sup></u>	<u>n</u>
ImpVol	92.564 (1.017)	0.307 (0.008)	0.178	6220
Trad	95.811 (1.137)	0.157 (0.007)	0.069	6220

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**Table II****Regression of CDS Spreads against Implied Credit Spreads, Firm by Firm and Day by Day**

Mean and median results for regressions of observed CDS spread against implied credit spread. The credit spreads are implied from ImpVol implementation and Trad implementation of Merton's (1974) model. The ImpVol implementation is the implementation we propose in equation (8) and (9). The Trad implementation is the traditional implementation of Merton's model in equations (1) and (2)). In the upper panel the regressions are time series regressions done on a firm-by-firm basis (86 regressions). In the lower panel the regressions are cross-sectional regressions done on a day-by-day basis (90 regressions). Only regressions with 30 or more observations are included.

<u>Implementation</u>		<u>Constant</u>	<u>Slope</u>	<u>R<sup>2</sup></u>	<u>N</u>
		Time Series by Ticker			
ImpVol	Mean	100.55	0.33	0.26	63
	Median	76.17	0.15	0.23	53
Trad	Mean	96.31	0.22	0.29	63
	Median	79.53	0.14	0.28	53
		Cross-section by Day			
ImpVol	Mean	90.95	0.88	0.24	37
	Median	92.68	0.36	0.18	35
Trad	Mean	97.17	0.40	0.12	37
	Median	99.37	0.21	0.06	35

**Table III**  
**Rank Order Correlation Measures**

Kendall's rank order correlation measure and Spearman's rank order correlation measure between the implied credit spread from Merton's (1974) model and the five-year credit default swap spread. The ImpVol implementation is the implementation of Merton's model we propose in equation (8) and (9). The Trad implementation is the traditional implementation of Merton's model in equations (1) and (2). The standard error of the rank order correlation is an upper bound. The  $z$ -statistic tests whether the rank order correlation is significantly greater than zero. The A – B rows use equation (A1) to test the difference between the A and B correlations.

	Kendall's Rank Correlation	Standard Error	$z$ -statistic	Spearman's Rank Correlation	Standard Error	$z$ -statistic
<b>All Data</b>						
ImpVol Implementation	0.2836	0.0172	33.55	0.4230	0.0199	33.36
Trad Implementation	0.2590	0.0173	30.63	0.3929	0.0202	30.99
ImpVol – Trad	0.0247	0.0244	1.01	0.0301	0.0284	1.06
<b>Firm-by-firm (<math>n = 86</math>)</b>						
ImpVol Implementation	0.3967	0.0188	38.01	0.5409	0.0202	35.51
Trad Implementation	0.3101	0.0193	28.71	0.4386	0.0212	28.79
ImpVol – Trad	0.0866	0.0270	3.21	0.1023	0.0293	3.49
<b>Day-by-day (<math>n = 90</math>)</b>						
ImpVol Implementation	0.2506	0.0239	20.37	0.3630	0.0280	20.35
Trad Implementation	0.2188	0.0241	17.78	0.3186	0.0285	17.87
ImpVol – Trad	0.0318	0.0340	0.94	0.0443	0.0400	1.11

**Table IV****Rank order correlations for different values of the debt maturity,  $T$** 

Spearman and Kendall rank order correlations between the credit spreads implied from Merton's model for different values of the debt maturity,  $T$  using the ImpVol implementation. The upper triangular portion of the matrix shows the Spearman correlation, the lower triangular portion shows the Kendall correlation.

	$T = 1$	$T = 2$	$T = 5$	$T = 10$
$T = 1$	1.000	0.978	0.954	0.988
$T = 2$	0.878	1.000	0.946	0.974
$T = 5$	0.826	0.800	1.000	0.947
$T = 10$	0.908	0.861	0.809	1.000

**Table V****Regression of observed CDS spread against the at-the-money volatility and the volatility skew**

A regression of observed CDS spread (CREDSPR) against the at-the-money volatility (ATMVOL) and the volatility skew (SKEW).

$$\text{CREDSPR} = a + b \times \text{ATMVOL} + c \times \text{SKEW}$$

SKEW is measured as the 25-delta volatility less the at-the-money volatility. Volatilities are measured as decimals and the CDS spreads are in basis points. Standard errors are shown in parentheses.

To address co-linearity issues the regressions are done first with SKEW orthogonalized with respect to ATMVOL and then with ATMVOL orthogonalized with respect to SKEW. \*\* Denotes significance at the 1% level. \*Denotes significance at the 5% level.

Restriction	Orthogonalized SKEW			Orthogonalized ATMVOL			$R^2$	$n$
	$a$	$b$	$c$	$a$	$b$	$c$		
None	-27.61** (3.68)	336.47** (8.84)	57.83 (36.86)	75.63** (1.81)	328.26** (10.27)	657.50** (31.72)	0.1889	6220
ATMVOL < 40%	7.62* (3.72)	222.96** (10.28)	14.13 (33.75)	72.31** (1.58)	220.68** (11.63)	313.74** (29.82)	0.1318	3088
40% < ATMVOL	-38.99** (7.62)	371.91** (16.45)	166.69* (66.95)	86.42** (3.48)	346.91** (19.28)	794.51** (57.15)	0.1413	3132

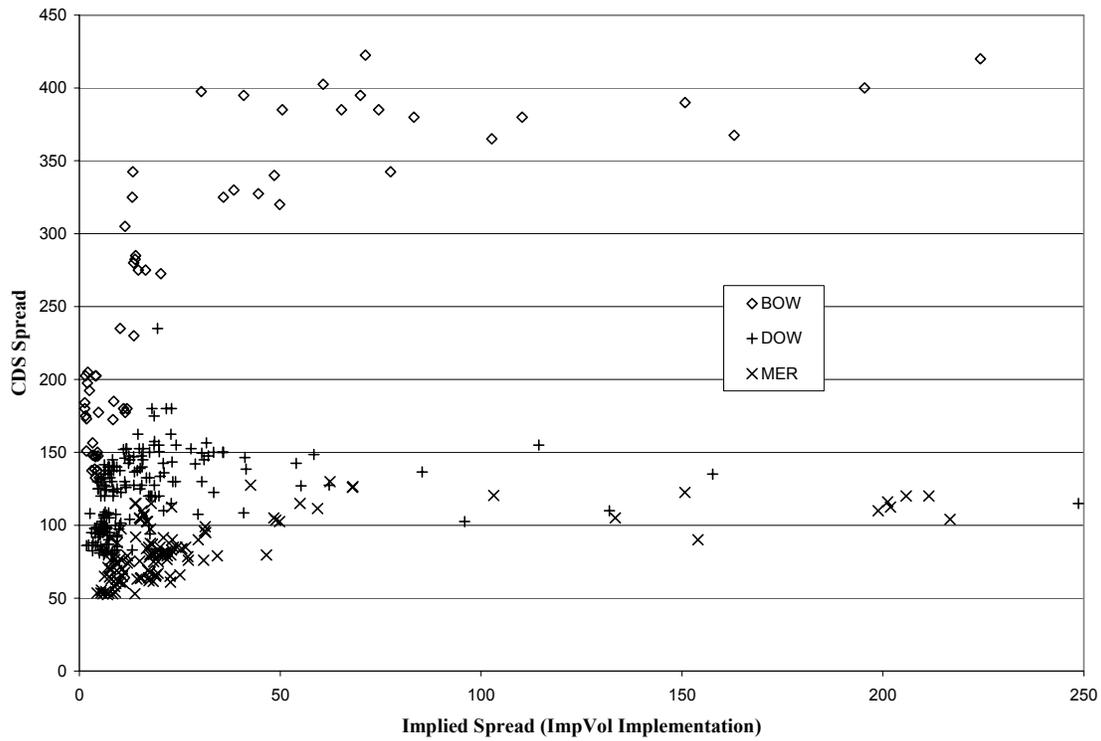
**Table VI**

**Comparison of Merton's (1974) model with a model based on Merton (1976) mixed jump diffusion process**

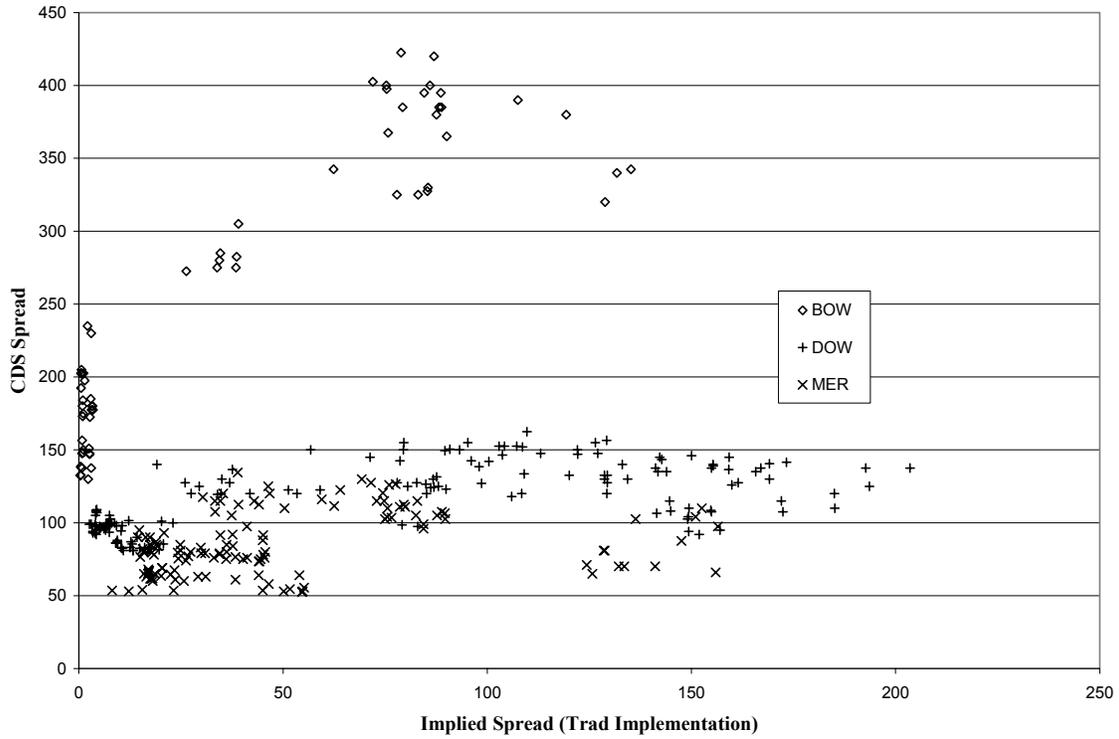
Data is pooled for all companies on all days The table shows the Kendall's rank order correlation measure and Spearman's rank order correlation measure between implied credit spread and the five-year credit default swap spread.

The standard error of the rank order correlation is an upper bound. The z-statistic tests whether the rank order correlation is significantly greater than zero. The last row uses equation (A1) to test the difference between the Merton (1974) and the Merton (1976) correlations.

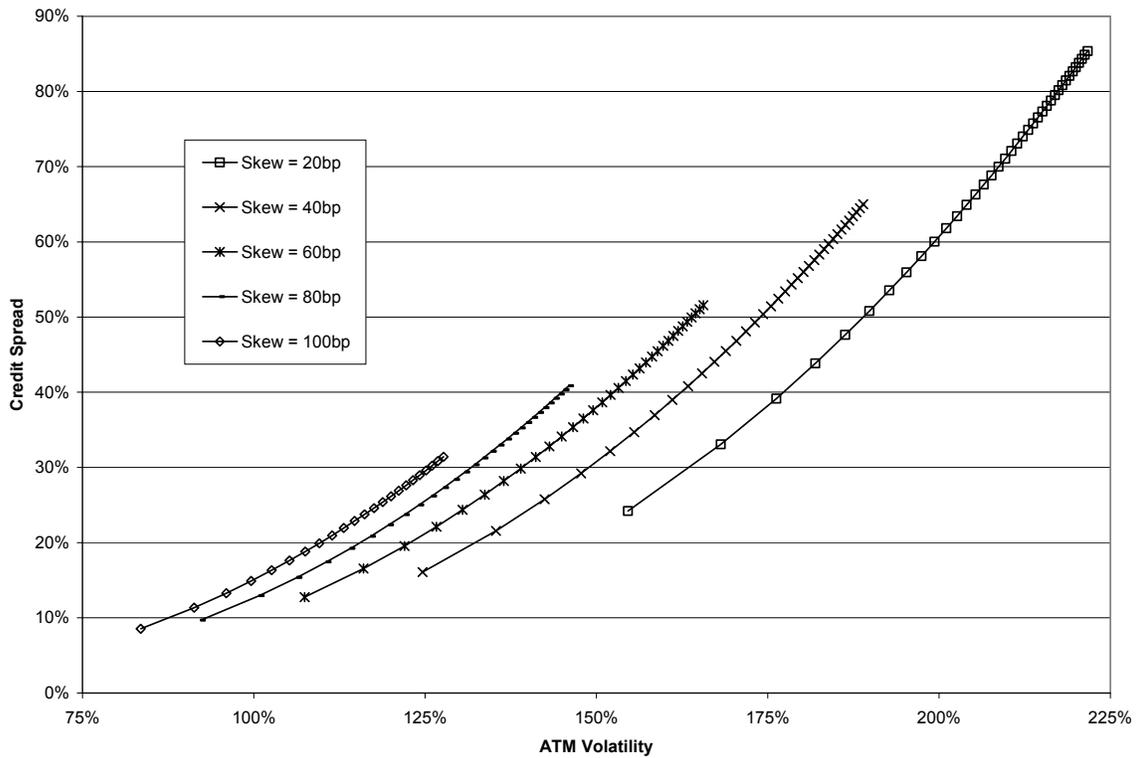
	Kendall's Rank Correlation	Standard Error	z-statistic	Spearman's Rank Correlation	Standard Error	z-statistic
Merton (1974)	0.2836	0.0172	33.546	0.4230	0.0199	33.362
Merton (1976)	0.2095	0.0175	24.778	0.3177	0.0208	25.056
Merton (1974) – Merton (1976)	0.0741	0.0246	3.019	0.1053	0.0288	3.657



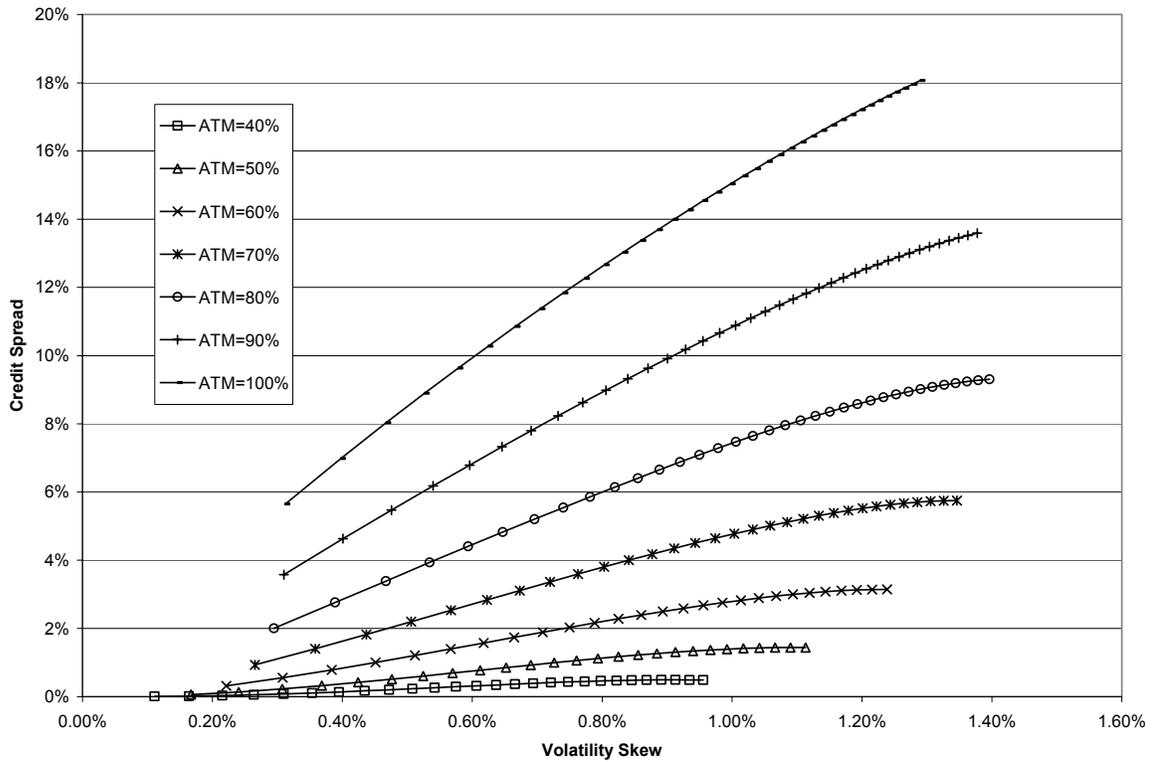
**Figure 1: Relationship between CDS spread and implied spread for ImpVol Implementation of Merton's Model.** The three companies are Merrill Lynch (MER), Bowater (BOW) and Dow Chemicals (DOW). The ImpVol implementation of Merton's model is the implementation we propose in equations (8) and (9).



**Figure 2: Relationship between CDS spread and implied spread for Trad implementation of Merton’s model.** The three firms are Merrill Lynch (MER), Bowater (BOW) and Dow Chemicals (DOW). The Trad implementation is the traditional implementation in equations (1) and (2)



**Figure 3: Theoretical relationship between credit spread and at-the-money volatility.** The relationship is implied by Merton's model for alternative values of the volatility skew when option maturity is two months and debt maturity is 5 years. The volatility skew is the difference between the volatility of an option with a delta of 0.25 and an option with a delta of 0.5.



**Figure 4: Theoretical relationship between credit spread and volatility skew.** Relationship is implied by Merton’s model for alternative at-the-money (ATM) volatilities when option maturity is two months and debt maturity is 5 years. The volatility skew is the difference between the volatility of an option with a delta of 0.25 and an option with a delta of 0.5.